Golden Root Geometry Structuring the Polyhedra and Other Forms Via Plato's Triangles

Quadrature of Circle

Eur Ing Panagiotis Chr. Stefanides BSc(Eng)Lon(Hons) MSc(Eng)NTUA TCG CEng MIET

Hellenic Aerospace Industry [ex] R&D, Aircraft Engines' Manufacturing Engineering Methods, Engineering Quality and Reliability, EMC
8, Alonion St., Kifissia, Athens-GR 14562
e-mail:panamars@otenet.gr , web : http://www.stefanides.gr

Abstract: Under Golden Root Geometry Structuring the Polyhedra and other Forms Via Plato's Triangles, we refer to the basic geometric configurations which, as this theory contemplates, are necessary for the progressive mode of formation of the five polyhedra, via lines, areas and volumes. Basis of all these structures is a very special Scalene Orthogonal Triangle "Plato's Most Beautiful" [F23], together with his Orthogonal Isosceles. Structural Forms are identified bearing in common these triangular identities. The particular angle of the Scalene Orthogonal is that whose ArcTan[Θ]=T and T = SQR ((SQR.(5) + 1)/2)

Keywords – The Most Beautiful Triangle, Orthogonal Scalene Triangle, Orthogonal Isosceles Triangle, Somatoides [F4] tetrahedral Structure, Great Pyramid Model [F8], Polyhedra, Circles Quadrature, Spirals, Sprialogarithm. [F15].

I.Introduction

By "Golden Root Geometry" we refer to two configurations of triangles. A Special one, the Quadrature Scalene Orthogonal Triangle [Author's interpretation of the Timaeic Most Beautiful Triangle] with sides [T^3], [T^2] and [T^1] in geometric ratio T, which is the square root of the golden ratio[Φ], and the Isosceles Orthogonal Triangle, with its equal sides [T]. The surface areas of these triangles are taken perpendicular to each other and in such, naturally, defining an X, Y, and Z system of coordinate axes. In so, the coordinates of the first are [0,0,0,], [0,0,T^2], [T,0,0] in the X-Z plane, and those of the second are [0,0,0,], [T,0,0], [0,T,0] in the X-Y plane. A line from [0,T,0] to [0,0,T^2],creates the same Scalene Triangle in the Y, Z plane.

Arctan [T] is the Scalene angle [θ] of the Special Triangle with the property that the product of its small side by its hypotenuse is equal to the square of its bigger side: [T^1]*[T^3] equal [T^2]^2 [Quadrature].

Using a pair of the Special Scalene Triangle, and a pair of a Similar Kind of Triangle [Constituent of the Special] with sides 1,T and T^2 [Kepler / (Magirus) Triangle with sides 1, sqrt(Φ), and Φ] a Tetrahedron [dicta Form 1] is


F1 [Lines]


F2 [Areas]


F3 [Volume]


F4 [Form 1]


F5 [Form 2]


F6 [Form 3]


F7


F8

obtained, by appropriately joining the edges of the four triangles, with coordinates: [0,0,0,], [0,0,T^2], [T,0,0] and [0, 1/T, 1/T^2]

By joining, a line, from point [0,T,0] to point [T,0,0], a Second Tetrahedron [dicta Form 2] is obtained [as a natural extension of Form 1], with co-ordinates: [0,0,0,], [T,0,0],[0,T,0] and [0,1/T, 1/T^2], having as base, on the X-Y plane, the Isosceles Orthogonal Triangle mentioned above, with coordinates[0,0,0], [T,0,0] and [0,T,0].
Doubling this triangle, in the X-Y plane, a square is obtained of side [T], with coordinates [T,T,0], [T,0,0], [0,0,0], and [0,T,0].

By connecting a line from point [T,T,0] to point [0,0,T^2] a third Tetrahedron [dicta Form 3] is obtained with coordinates:[T,T,0],[T,0,0], [0,T,0] and [0,0,T^2], having also as base, the Isosceles Orthogonal Triangle with same dimensions [mirror image] as that of [Form 2]. The three Forms are wedged firmly together, leaving no empty space between them. Their volume ratios Form 3: Form 1: Form 2 equal to [1/6]*[T*T*T^2]: [1/6]*[1*T*T]: [1/6]*[T*T*(1/T^2)] is the golden ratio [T^2], and the sum of volumes of Form 1 and Form 2 equals to [1/6]*[1*T*T] + [1/6]*[T*T*(1/T^2)] equals to [1/6]*[T^2+1] equal [1/6]*T^4[SINCE T^4-T^2-1=0], the volume of Form 3.The volumes of the three Forms sum up to [(2/6)T^4 equal to (1/3)T^4].
Two of the four bases of Form 3, are symmetrical orthogonal triangles, with coordinates [T,0,0], [T,T,0], [0,0,T^2] and [T,T,0], [0,T,0], [0,0,T^2], each of which has an angle [φ], equal to arctan[T^2].
Two such triangles joined in a coplanar manner, and symmetrically along their bigger vertical sides, create one of the four triangular faces of a great pyramid model with coordinates [T,T,0], [0,0,T^2] and [T,(-T),0].

The Structure of the three Forms bound together [dicta Form 4] with Volume [1/3]*T^4 is one quarter of the volume of the great pyramid model, which has a square base of side 2T, height T^2 and Volume [4/3]*T^4.

Splitting one of this model's triangular face into two orthogonal co-planar triangles to form a parallelogramme [with sides T^1 and T^3], we have constructed the basic skeleton of the Icosahedron [F11], since three such parallgrammes, orthogonal to each other, determine its twenty equilateral bases, by joining adjacent corners in groups of three, by lines.

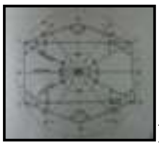
Similarly, we proceed to the construction of the dodecahedron, the tetrahedron, the octahedron and the cube, together with their related forms such as squares, circles, triangles, circumscribed circles to the parallelogrammes of the polyhedra skeletons, circumscribed spheres and logarithmic spirals.


F9


F10


F11


F12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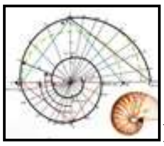
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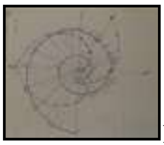
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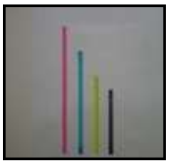
F16


F17

Reversing the whole process, the volumes decompose to the areas of the triangle surfaces structuring them which, in turn, ***resolve*** to ***four line traces harmonically codified in space*** [F17].

II. Platonic Timaeus Triangles

The work as described above follows, according to my interpretation of Plato's Timaeus description of "THE MOST BEAUTIFUL TRIANGLE" and further, basing on this, the structure of his "world" of his Polyhedra. Lines of triangles represent elements [combinations of the 4 philosophical ones: Fire, Air, Earth and Water].Solods created bear the same names, but include a further "Consistency" according to Plato for the Solid "Aether".

IIA. Sections 53-54 of Timaeus

According to Plato's Timaeus,

…."The conditions prevailed before the World was Created, while all elements [FIRE, AIR, EARTH and WATER] were "WITHOUT PROPORTION" [alogos] and "WITHOUT MEASURE" [ametros], and only "TRACES" of them existed, as all things, naturally exist in God's absence. God, under these conditions, transformed them via "IDEAS" and "NUMBERS", for them to become "MOST BEAUTIFUL" and "BEST" as possible, contrary to their previous state…..

…. Πρώτον μεν δη πυρ και γη και ύδωρ και αήρ, ότι σώματά εστί.. τρίγωνα πάντα εκ δυοίν άρχεται τριγώνοιν.... προαιρετέον ούν αύ των απείρων το ΚΑΛΛΙΣΤΟΝ..... ΤΡΙΠΛΗΝ ΚΑΤΑ ΔΥΝΑΜΙΝ ΕΧΟΝ ΤΗΣ ΕΛΑΤΤΟΝΟΣ ΤΗΝ ΜΕΙΖΩ ΠΛΕΥΡΑΝ ΑΕΙ"…….

In sections 53-54, of Plato's "Timaeus", Plato speaks about the triangular shapes of the Four Elements [traces existed in dissorder –matter- before their harmonization by God], of their kinds and their combinations:

These Bodies are the Fire (Tetrahedron) the Earth (Cube), the Water (Icosahedron), and the Air (Octahedron). These are bodies and have depth. The depth necessarily, contains the flat surface and the perpendicular to this surface is a side of a triangle and all the triangles are generated by two kinds of orthogonal triangles: the "Isosceles" Orthogonal and the "Scalene" Orthogonal. From the two kinds of triangles the "Isosceles" Orthogonal has one nature. (i.e. one rectangular angle and two acute angles of 45 degrees), whereas the "scalene" has infinite (i.e. it has one rectangular angle and two acute angles of variable values having, these two acute angles, the sum of 90 degrees). From these infinite natures we choose one triangle "The Most Beautiful". Thus, from the many triangles, we accept that there is one of them "The Most Beautiful». Let us choose then, two triangles, which are the basis of constructing the Fire and the other Bodies : "Το μεν ισοσκελές,το δε τριπλήν κατά δύναμιν έχον της ελάττονος την μείζω πλευράν αεί."

IIB. Proposed New Interpretation:

One of these two is the "Isosceles" Orthogonal Triangle, the other is the "Scalene" Orthogonal Triangle, its hypotenuse having a value equal to the "Cube" of the value of its horizontal smaller side and having its vertical bigger side the value of the "Square" of its smaller horizontal side. The value of the smaller horizontal side is equal to the square root of the Golden Number, the ratio of the sides is equal, again, to the Square Root of the Golden Number (geometrical ratio) and the Tangent of the angle between the hypotenuse and the smaller horizontal side is also equal to the Square Root of the Golden Number (Θ =51 49-38-15-9-17-19-54-37-26-24-0 degrees). The product of the smaller horizontal side and that of the hypotenuse is equal to the "SQUARE" of the bigger vertical side, and the following equation holds: T^4-T^2-1=0 , T = SQRT [(SQRT(5) + 1)/2]. The Kepler [Magirus] Triangle is a similar one but not the same. By "THE MOST BEAUTIFUL TRIANGLE", Plato correlates the four elements (UNIFIED THEORY) through the General Analogies of their sides (Fire, Air, Earth and Water), i.e. Fire/Air is equal to Air/Water is equal to Water/Earth, to T, where T is equal to the SQUARE ROOT of the GOLDEN NUMBER: T = SQR ((SQR.(5) + 1)/2)

(ό τι περ πύρ προς αέρα, τούτο αέρα προς ύδωρ, και ό τι αήρ προς ύδωρ, ύδωρ προς γήν, ξυνέδησε...............ουρανόν, Plato's Timaeus section 32).

IIC. Section 37 - 39 of Timaeus

According to Plato [Timaeus 37 -39] :

...He planned to make a movable image of Eternity, He madean eternal image, moving according to number, even that which we have named Time.... He contrived the production ofdays and nights and months and years ...And these are allportions of Time; as "Was" and "Shall be" are generated forms of Time...Time, then, came into existence along withHeaven, to the end that having been generated together theymight also be dissolved together...this reasoning and design...with a view to the generation of Time, the sun and moon....and the other stars,....”planets”, came into existence for the determining and preserving of the numbers of Time..... [Loeb].

Trying to understand the way possible for elements [according to Plato: traces - i.e. matter] to be joined together so that they compose matter, according to the geometry presented in this paper, we come to the following, possible, scenario:

In a static, but vibrating, field [aether- Plato's reference to a fifth Consistency, which God used up entirely to " colourfully paint " everything i.e. means by which matter is illuminated reflecting light from source (Fire) - electromagnetic medium], conductive [massive] elemental lines with alternating bipolar charges moving in it by the action of the field , should result into alternating currents running within them. Two such lines could be contacted electrically at the ends of each line, via their + and - charges, and similarly three lines [in the correct lengths] could form triangles [orthogonal according to my theory], and in such forming a surface. Similarly by joining two pairs of such triangular forms [elecromagnetically attracted by the currents running within them] could create materialistic volumes[tetrahedra].Continuing, by these similar actions of electromagnetic forces, the joining of these materialistic volumes[tetrahedra] could result into further building blocks of matter.
According to my geometric theory [pure classical geometry, based on the Square Root of the Golden Section] such materialistic volumes [tetrahedra] build a Great Pyramid Model via which the structure of the world of the 5 Platonic [or Eucleidean] solids are formed.

We note that Plato states that everything that is born, it is born by necessity due to a cause, because it is impossible for it to be born without a cause.
With respect to world's creation, Plato states that, according to his thinking after having performed some assumptions, that three things exist before Heaven's coming into creation, Being, Place and Becoming.

III. Circle’s Quadrature [F18, F20, F21]

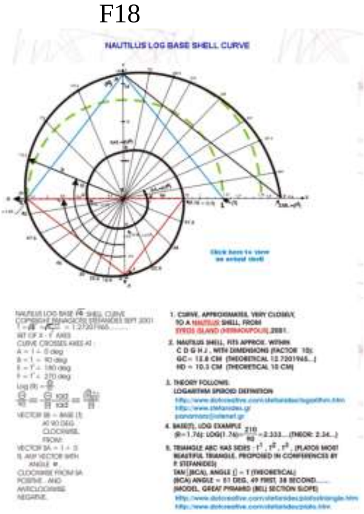
Structures built upon the form of the two Orthogonal Triangles, as described in this paper, have common relationships i.e. triangles, parallelograms, squares, circles.

The main classical problem concerning circle relationships with the other geometric figures was that of its quadrature [conditions finding for its area to be equal to that of a square and its circumference equal to the perimeter of another square].

Plato’s Scalene Orthogonal Triangle after lengthy elaboration, worked catalytically for forming a novel concept, for achieving the challenge of solving the “ insoluble” problem of “quadrature of circle” [F , F , F], Proving that the value of pi should be quantized : PI =4/[SQRT(Phi)], i.e pi equals 4 divided by the square root of the Golden Section [= 3.14460551…..].

Further a [PCST] Point on the Circle, the Square ,the Triangle] Maximum Symmetry Point [F22], was geometrical conceived for demonstration that the value of Pi should be quantized to the value of : 4/[SQRT(Phi)], as achieved ,also, by ruler and compass [F18].
This serves as a gauge for estimating errors [shifts from PCST] of Pi values, used diachronically.

F18



F19

[F18 , F19, F20] AutoCAD Drawings
Vector Definition and Geometry Design by Panagiotis Stefanides , Computerized AutoCAD by Dr. John Candylas

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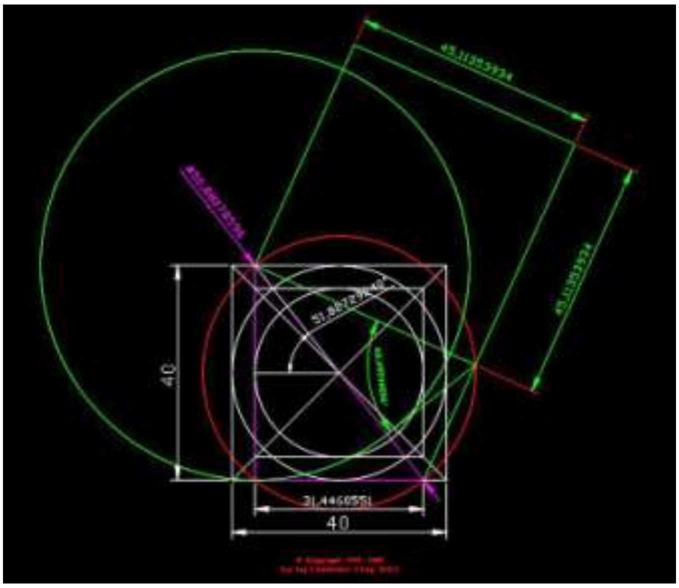
III. Conclusions

Via the Golden Root Geometry we get relationships of Geometric structures, Logarithms and Spirals.
It is concluded that by "THE MOST BEAUTIFUL TRIANGLE", Plato correlates the elements (Unified Theory) through the general analogies of their sides i.e. Fire/Air is equal to Air/Water is equal to Water/Earth, is equal to T the Golden Root.

Finally, we realize Plato's statement that all triangles derive from two ORTHOGONAL TRIANGLES the ISOSCELES and the SCALENE.



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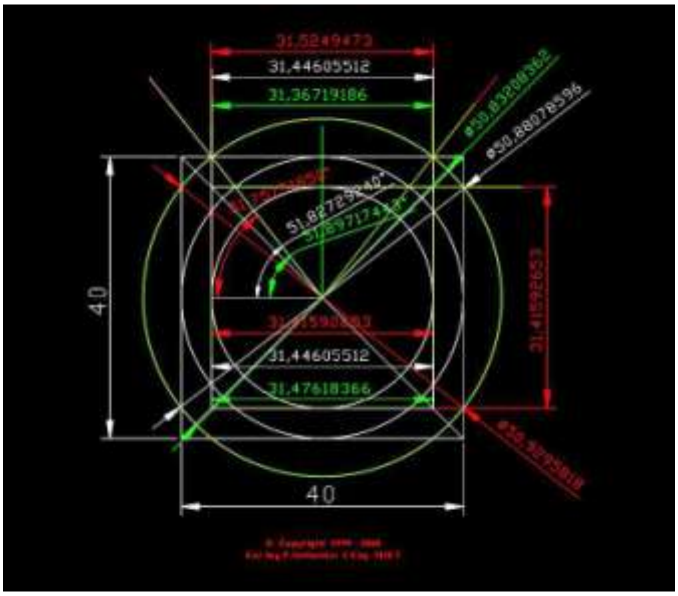

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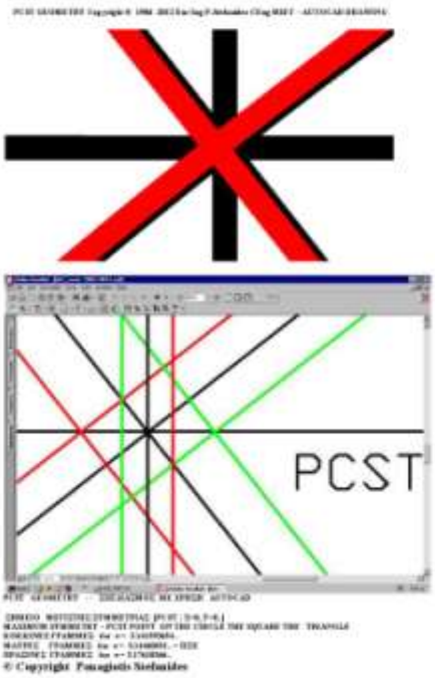

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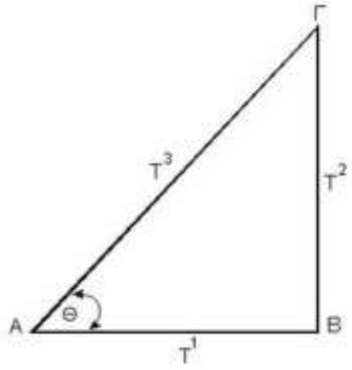
F23



$$T^4 - T^2 - 1 = 0$$

$$T^6 - T^4 - T^2 = 0$$

$$T^6 = T^4 + T^2$$

$$(T^3)^2 = (T^2)^2 + (T^1)^2$$

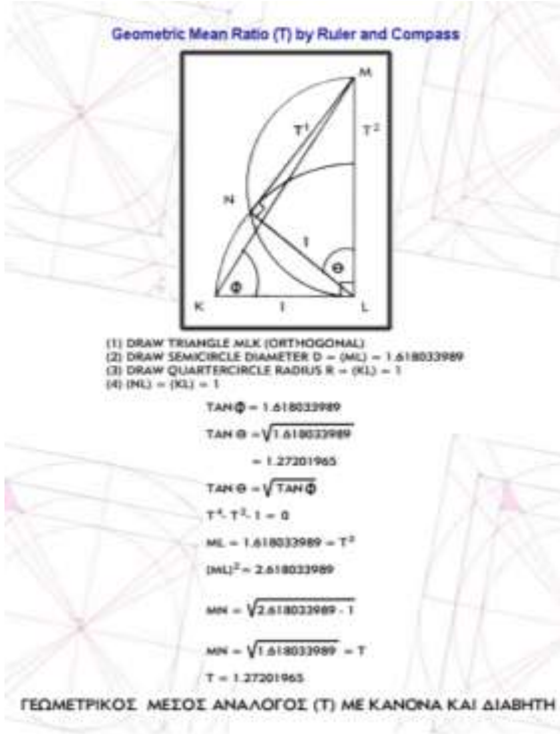
$$(A\Gamma)(AB) = (\Gamma B)^2$$

$$\text{TAN } \Theta = \frac{T^2}{T^1} = T$$

$$\Theta = \text{TAN}^{-1}(T)$$

$$\frac{A\Gamma}{\Gamma B} = \frac{\Gamma B}{BA} = T$$

F24



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Biography

Eur Ing Panagiotis Chr. Stefanides BSc(Eng) Lon(Hons) MSc(Eng)Ath TEE CEng MIET, Emeritus Honoured Member of the Technical Chamber of Greece

Born: 05. Jan..1945, Aegaleo, Athens.

Professional Academic Qualifications and Experience:

[2002] Chartered Engineer of the Engineering Council (UK),

[2010] IET[IEE] Hellas Nerwork Honorary Secretary

[1977] Member of the Technical Chamber of Greece TEE,
[1975] Electrical and Mechanical Engineer NTUA
[1974] Electrical Engineer of the University of London .
[1978] - [2010] HAI, Including assignments such as R&D Electronic Systems, HAI-SNECMA,. M53P2 Aircraft Engine Nozzle Coproduction Manufacturing Programme.